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De Sitter vacua from $N = 2$ gauged supergravity

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Abstract: Typical de Sitter (dS) vacua of gauged supergravity correspond to saddle points of the potential and often the unstable mode runs into a singularity. We explore the possibility to obtain dS points where the unstable mode goes on both sides into a supersymmetric smooth vacuum. Within $N = 2$ gauged supergravity coupled to the universal hypermultiplet, we have found a potential which has two supersymmetric minima (one of them can be flat) and these are connected by a de Sitter saddle point. In order to obtain this potential by an abelian gauging, it was important to include the recently proposed quantum corrections to the universal hypermultiplet sector. Our results apply to four as well as five dimensional gauged supergravity theories.

Keywords: Nonperturbative Effects, Superstrings and Heterotic Strings, Superstring Vacua, Supergravity Models.

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1. Introduction

The conjectured AdS/CFT duality [1] has led to renewed intense research in gauged supergravity theories in various dimensions. The study of $N = 2$ gauged supergravity theories has particularly enriched our understanding regarding many interesting aspects of domain wall solutions and the associated renormalization group (RG) flow in a dual superconformal field theory [2, 3]. These domain wall solutions have also played an important role in the context of brane world scenario proposed as an alternative to compactification [4].

Type-II or M-theory when compactified on a Calabi-Yau threefold gives rise to $N = 2$ models with vector and hypermultiplets in four and five dimensions respectively. The scalars belonging to the vector and hypermultiplets are known to parametrize a manifold that is a product of a special Kähler and a quaternionic manifold. The various gauging procedures involving gauging of the abelian $\mathrm{U}(1)_R$ symmetry, the full $\mathrm{SU}(2)_R$ symmetry, isometries of the vector and the hypermultiplets [5]–[10] provide an explicit derivation of the superpotential and the associated scalar potential. From the analysis of the critical points of the potential, one has several instances where the potential admits supersymmetric extrema. Explicit domain wall solutions have been obtained which interpolate between different extrema of the potential. In order to make contact with string or M-theory, one is especially interested in potentials that are related to flux or Scherk-Schwarz reductions (or massive T-dualities) [11].

Besides supersymmetric extrema, the potentials can also exhibit de Sitter extrema so that supersymmetry is spontaneously broken in these vacua. The exploration of de Sitter vacua in string or M-theory has attracted much attention in recent times, not only from the point of view of gauged supergravity (or hyperbolic reductions) [12]–[17], but also specific brane configurations have been investigated from various perspectives [18]–[20].

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The potential obtained by explicit supersymmetry breaking (e.g. by addition of anti branes) have more room for phenomenologically interesting quantities as the strongly constrained potentials obtained in gauged supergravity. For example, de Sitter vacua in gauged sugra typically correspond to saddle points (with some notable exceptions as e.g. in [16]) and often do not fulfil the slow roll requirement [15, 21], which ensures sufficient inflation. Therefore, these potentials can only contribute to fast roll inflation as proposed in [22], which might nevertheless occur during some period of the cosmological evolution. On the other hand, a saddle point can give a so-called locked inflation, where the scalar field oscillates along the stable direction with an amplitude much larger than the curvature scale of the unstable direction [23], see also [24].

An especially rich vacuum structure in gauged supergravity is expected to arise if one gauges the isometries of the quantum corrected moduli space. Only very few quantum corrections have been explicitly calculated. In some cases the correct geometry of these spaces follows by imposing lower dimensional supersymmetry combined with certain number of isometries respected by the quantum corrections [25]–[28]. For example in the double-tensor calculus [29], the gauge symmetries ensure that the 4-dimensional quaternionic space of the universal hypermultiplet (UH) has at least two commuting isometries. The explicit construction of these spaces is a fairly challenging task, but fortunately, the most general four dimensional quaternionic metric with a $U(1) \times U(1)$ isometry group has been found by Calderbank and Pedersen [30], which encodes all perturbative and non-perturbative corrections that respect this symmetry. Besides a non-perturbative instanton sum, this metric includes the 1-loop correction to the universal hypermultiplet moduli space, which has been recently verified by explicit calculation in the nice paper [27], that moreover shows that all higher loop corrections correspond to field redefinitions and that the instanton sum has to be related to D2/M2-brane instanton, see also [28]. Additional D4 or 5-brane instantons would break all isometries and may leave only discrete symmetries and hence are not described by this metric.

Therefore, the investigation of this moduli space from the point of view of gauged supergravity, opens the window of understanding of non-perturbative physics in the vacuum structure. Supersymmetric vacua have been investigated already in [8] and we want to focus in this paper on possible de Sitter vacua coming from this metric. To keep the expressions as simple as possible we consider only this universal hypermultiplet and couple it the $N = 2$ supergravity. In our analysis we do not consider any vector multiplets and therefore our results apply to four as well as five dimensional $N = 2$ supergravity, as in both cases the hypermultiplet moduli space is given by a quaternionic space. But of course, from cosmological point of view, one might prefer to use our results in the 4 dimensions and in most applications we implicitly assume to be in 4 dimensions.

We shall gauge a linear combination of the two abelian isometries and find besides the supersymmetric extrema, a saddle point of the potential that corresponds to a de Sitter vacuum. The unstable mode flows on both sides to supersymmetric vacua and for a proper choice of parameter one of them is flat and the other one is anti de Sitter. Since there is no supersymmetric flow between two flat space vacua, the de Sitter saddle point has to have an AdS vacuum on one side. Note, the absence of vector multiplets implies that the BPS vacua should not break supersymmetry, because the breaking of $N = 2$ to $N=1$ supersymmetry requires a massive spin 3/2 multiplet which has to contain *two* vectors and therefore requires at least one vector multiplet, see e.g. in [31]. But because this metric has two commuting isometries it is straightforward to gauge one of them with the graviphoton and the second one with a vector of an additional vector multiplet.

The organization of the paper is as follows: in section 2, we summarize relevant aspects of $N = 2$ gauged supergravity and quaternionic spaces, in section 3, we discuss the Calderbank-Pedersen metric and its relation to quantum corrected universal hypermultiplet moduli space. Section 4 contains the details of the gauging, the superpotential and the analysis of the critical points. In section 5, we discuss the cosmological flow from the de Sitter saddle point to the flat space minimum and finally in section 6 we present a short summary and outlook.

2. $N = 2$ gauged supergravity and quaternionic spaces

The field content of the bosonic sector of $N = 2$ supergravity coupled to n_H hypermultiplets are the graviton e_μ^a, the graviphoton A_μ and the $4n_H$ hyperscalars q^m. We will primarily be interested in the 4-dimensional case, but our results will also hold in 5 dimensions since we do not consider couplings to vector multiplets. Moreover, if we ignore the graviphoton, which will play no role here, the bosonic part of the lagrangian is then given by,

$$e^{-1}\mathcal{L}^{N=2}_{\text{bosonic}} = -\frac{1}{2}R - \frac{1}{2}g_{mn}\nabla_\mu q^m \nabla^\mu q^n - g^2 V\,, \tag{2.1}$$

where, the scalar potential V reads as,

$$V = -3\,P^i P^i + 2\,k^m k^n g_{mn}(q) \tag{2.2}$$

and the covariant derivative of the hypermultiplet scalars is given by,

$$\nabla_\mu q^m = \partial_\mu q^m + g A_\mu k^m(q)\,. \tag{2.3}$$

In the above, g_{mn} is the metric of the 4 dimensional quaternionic space, $k^m(q)$ are the Killing vectors of the gauged isometries of the quaternionic space with the gauge coupling constant g and $P^i(q)$ $(i = 1, 2, 3)$ are the corresponding triplet of Killing prepotentials.

Quaternionic-Kähler spaces allow for three (almost) complex structures J^i $(i = 1, 2, 3)$ defined by the algebra,

$$J^i \cdot J^j = -\delta^{ij} + \epsilon^{ijk} J^k\,. \tag{2.4}$$

Denoting the quaternionic vielbein by e^m, one obtains the triplet of 2-forms Ω^i as,

$$\Omega^i = -\frac{\kappa}{2} e^m \wedge J^i_{mn} e^n\,, \tag{2.5}$$

where, $m, n = 1, 2, 3, 4$ are the quaternionic indices. The holonomy group of a $4n$ dimensional quaternionic manifold is contained in $Sp(1) \times Sp(n)$. For $n = 1$ this statement can be replaced by the requirement that the Weyl-tensor of a 4-dimensional quaternionic space has to be (anti)-self-dual i.e.

$$W + \star W = 0\,. \tag{2.6}$$

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For a quaternionic space in any dimension, the triplet of 2-forms Ω^i is expressed in terms of the SU(2)-part of the quaternionic connection A^i (not to be confused with the graviphoton!)

$$dA^i + \frac{1}{2}\epsilon^{ijk} A^j \wedge A^k = \Omega^i \tag{2.7}$$

which ensures that the triplet of two forms are covariantly constant with respect to the SU(2) connection, i.e.

$$d\Omega^i + \epsilon^{ijk} A^j \wedge \Omega^k = 0 \tag{2.8}$$

and can be obtained from the spin connection 1-form w^{mn} of the quaternionic space

$$A^i = \frac{1}{2} w^{mn} J^i_{mn} \,. \tag{2.9}$$

In the same way, the self-dual part gives the $Sp(n)$ connection.

Any quaternionic space is Einstein and as hypermultiplet moduli space, it has to have negative scalar curvature and three complex structures, which are usually choosen as anti-self-dual ($J^i_{mn} = -\frac{1}{2}\epsilon_{mnpq} J^i_{pq}$) so that the triplet of 2-forms can be written as,

$$\begin{aligned} \Omega^1 &= (e^1 \wedge e^4 - e^2 \wedge e^3) \,, \\ \Omega^2 &= (e^1 \wedge e^3 + e^2 \wedge e^4) \,, \\ \Omega^3 &= (-e^1 \wedge e^2 + e^3 \wedge e^4) \,, \end{aligned} \tag{2.10}$$

where e^m are the vielbeine for the quaternionic metric g_{mn}. The isometries of this quaternionic manifold are generated by the Killing vectors $k^m_I(q)$

$$\delta q^m = \epsilon^I k^m_I(q) \,. \tag{2.11}$$

In component notation, the Killing vectors can be expressed in terms of the SU(2) triplet of Killing prepotentials $P^i_I(q)$ as

$$k^n_I \Omega^i_{mn} = \nabla_m P^i_I = \partial_m P^i_I + \epsilon^{ijk} A^j P^k_I \,, \tag{2.12}$$

where, $i = 1, 2, 3$ are the SU(2) indices, m, n are the quaternionic indices and I labels different isometries. One can write the prepotentials as (we refer to the appendix of [10] for more technical details)

$$P^i_I = \nabla_m (k_I)_n (\Omega^i)^{mn} \,. \tag{2.13}$$

With these prepotentials, we introduce the scalar superpotential W by,

$$W = \sqrt{P^i P^i} \tag{2.14}$$

such that the potential (in the real notation) can be expressed in the form,

$$V = 3\left(\frac{2}{3} g^{mn} \partial_m W \partial_n W - W^2\right) . \tag{2.15}$$

3. Universal hypermultiplet and Calderbank-Pedersen metric

The universal hypermultiplet arising in every Calabi-Yau compactification of type IIA/M-theory e.g., contains the dilaton ϕ, the axion D and a complex Ramond-Ramond scalar field C. The corresponding moduli space receives both a 1-loop as well as non-perturbative quantum corrections. In this section, we want to consider the quantum corrected moduli space.

Classically, the universal hypermultiplet parametrizes the quaternionic-Kähler space $\frac{\mathrm{SU}(2,1)}{\mathrm{U}(2)}$, which is a symmetric coset space and is parametrized by the complex NS-NS and R-R fields S and C respectively or alternatively in terms of four real scalars. In type-IIA theory, the real part of the NS-NS field S is related to the four dimensional dilaton and the imaginary part is related to the axion D which is dual to the three form H. Loop corrections to the universal hypermultiplet has also been discussed earlier [25]. The recent calculation of ref.[27] shows that nontrivial perturbative quantum corrections appear only at the 1-loop order and render the space non-kählerian. They have been obtained by considering possible deformations of the quaternionic target space metric which preserve the Heisenberg subgroup of the SU(2,1) isometry group of classical moduli space. This subgroup is generated by the classical Peccei-Quinn shift symmetries, which should not be broken at the perturbative level. Moreover, they have shown that all higher loop corrections can be absorbed into field redefinitions.

This immediately raises the question about non-perturbative corrections, which in general is a notoriously difficult question. But one may start with a subclass of deformations which leave invariant certain symmetries as e.g. the U(1) × U(1) subgroup, which appears e.g. as gauge symmetry in the double tensor calculus [29]. Assuming this symmetry, Calderbank-Pedersen were able to find the most general (quaternionic) metric [30], which reproduces exactly the 1-loop corrections found by Antoniadis etal. [27]. They moreover argue that further corrections represent a summation over all D2/M2 instantons (in the type IIA/M-theory setting); see also [28] for related proposals. In addition, if one also includes D4- and 5-brane instantons, one cannot expect the resulting moduli space to have any isometries. This however, would exclude any gauging of isometries and therefore the hyperscalars do not enter the potential and it is then unclear how one can fix these moduli (note however, due to back reaction the corresponding internal space may not have the moduli that need to be fixed).

The anti selfdual Einstein metric of Calderbank and Pedersen (CP) can be written in the form [30],

$$ds^2_{CP} = \frac{F^2 - 4\rho^2(F_\rho^2 + F_\eta^2)}{4F^2}\left(\frac{d\rho^2 + d\eta^2}{\rho^2}\right) + \\ + \frac{[(F - 2\rho F_\rho)\alpha - 2\rho F_\eta \beta]^2 + [(F + 2\rho F_\rho)\beta - 2\rho F_\eta \alpha]^2}{F^2[F^2 - 4\rho^2(F_\rho^2 + F_\eta^2)]}\,, \tag{3.1}$$

where, the 1-forms α and β are

$$\alpha = \sqrt{\rho}d\phi\,, \qquad \beta = \frac{(d\psi + \eta d\phi)}{\sqrt{\rho}} \tag{3.2}$$

and the two commuting isometries are given by the Killing vectors: ∂_ϕ and ∂_ψ. In another form, the same metric becomes

$$ds^2_{CP} = \frac{1}{F^2(\rho,\eta)} \left[\det Q \, \frac{d\rho^2 + d\eta^2}{\rho^2} + \frac{1}{\det Q} (d\phi \, , \; d\psi) \, N^t Q^2 N \begin{pmatrix} d\phi \\ d\psi \end{pmatrix} \right] \tag{3.3}$$

with

$$Q = \begin{pmatrix} \frac{1}{2}F - \rho\partial_\rho F & -\rho\partial_\eta F \\ -\rho\partial_\eta F & \frac{1}{2}F + \rho\partial_\rho F \end{pmatrix} , \qquad N = \frac{1}{\sqrt{\rho}} \begin{pmatrix} \rho & 0 \\ \eta & 1 \end{pmatrix} . \tag{3.4}$$

This metric has positive scalar curvature if $\det Q > 0$. For $\det Q < 0$, $(-g_{CP})$ is an anti-selfdual Einstein metric with negative scalar curvature $R(-g_{CP}) = -12$. The point where $\det Q = 0$ is a curvature singularity whereas a zero of F is a conformal infinity (end of a given coordinate space). As we will see below, one can find a choice of parameter so that the curvature singularity, that separates the positively and negatively curved quaternionic space, is not there in a given coordinate region.

The metric is completely specified by the function $F(\rho, \eta)$, which is a real function of two variables η and ρ and obeys the equation

$$\rho^2 (\partial_\rho^2 + \partial_\eta^2) \, F(\rho,\eta) = \frac{3}{4} \, F(\rho,\eta) \tag{3.5}$$

i.e. it is an eigenfunction of the two dimensional Laplace-Beltrami operator for the hyperbolic metric $\frac{d\rho^2+d\eta^2}{\rho^2}$ with eigenvalue $\frac{3}{4}$. This linear equation is invariant under $\mathrm{SL}(2,\mathbb{R})$ transformation of

$$\tau = \eta + i\,\rho \quad \to \quad \frac{a\tau + b}{c\tau + d} \quad , \qquad ad - bc = 1 \tag{3.6}$$

and hence the most general solution is given by a modular function, as discussed below.

If we first ignore the η-dependence, the basic solutions to the eigenvalue equation are given by power functions

$$F_s(\rho,\eta) = \rho^s \, , \qquad s = \frac{3}{2} \, , \; -\frac{1}{2} \, . \tag{3.7}$$

With the field identifications

$$\rho^2 = e^{-2\phi} \, , \qquad C = C_1 + iC_2 = \eta + \frac{i}{2}\phi \, , \qquad \psi = D - C_1 C_2 \tag{3.8}$$

one can check that $F(\rho,\eta) = \rho^{3/2}$ gives exactly the tree level result, i.e. the CP metric reduces the known classical universal hypermultiplet metric. As shown in [27] the 1-loop correction to this moduli space can be reproduced by adding both solutions:

$$F(\rho,\eta) = \rho^{3/2} - \hat{\chi}\,\rho^{-1/2} \, . \tag{3.9}$$

The explicit calculation yields: $\hat{\chi} = -\frac{4\zeta(2)\chi}{(2\pi)^3}$, where χ is the Euler number of the Calabi-Yau space and due to this 1-loop correction, the metric becomes non-Kähler. Moreover it has a singularity at $\rho^2 = \hat{\chi}$ where $\det Q = 0$. Note, the field redefinition (3.8) receives also corrections [27] so that the singularity appears at strong coupling $e^{-2\phi_4} \to \infty$. At this singularity we cannot trust anymore the approximation and have to include further corrections, i.e. we have to take into account the η-dependence of F.

This is done by the basic solutions to the eigenvalue equation, which is given by,

$$F^{(m,n)}(\rho,\eta) = \left[\frac{\mathrm{Im}\,\tau}{|m\tau+n|^2}\right]^s = \left[\frac{\rho}{m^2\rho^2+(m\eta+n)^2}\right]^s , \qquad s = \frac{3}{2}\,,\ -\frac{1}{2}\,. \tag{3.10}$$

For $m = 0$ both the functions obviously reduce to the previous case and they are invariant under $\mathrm{SL}(2,\mathbb{Z})$ transformations as given in eq. (3.6) where (m,n) transform as

$$\begin{pmatrix} m \\ n \end{pmatrix} \to \begin{pmatrix} a & c \\ b & d \end{pmatrix} \begin{pmatrix} m \\ n \end{pmatrix} .$$

Note, the equations are in fact $\mathrm{SL}(2,\mathbb{R})$ invariant, but only for $\mathrm{SL}(2,\mathbb{Z})$ one can make a relation to an instanton sum. We see that the $\mathrm{SL}(2)$ transformations map the different solutions into each other and summing over all co-prime integers (m,n) gives the modular invariant Eisenstein series E_s, which has been interpreted as a summation over all (anti) instantons in [32, 28] and appears in the 4-d string frame action ($S_4 \sim \int \sqrt{\rho} E_{3/2}[R + \cdots]$) [27]. Using the formula: $\Lambda(s)E_s = \Lambda(1-s)E_{1-s}$ with $\Lambda(s) = \pi^{-s}\Gamma(s)\zeta(2s)$ [33], both summations with $s = 3/2$ or $s = -1/2$ are equivalent. The classical limit is obviously related to the value $s = 3/2$, which has as asymptotic expansion for large ρ

$$\begin{aligned} 4\pi E_{3/2}(\rho,\eta) \;&= 2\zeta(3)\rho^{3/2} + \frac{2\pi^2}{3}\rho^{-1/2} + 4\pi^{3/2}\sum_{m,n\geq 1}\left(\frac{m}{n^3}\right)^{1/2} + \\ &\quad + \left[e^{2\pi i m n(\eta+i\rho)} + e^{-2\pi i m n(\eta-i\rho)}\right]\left[1+\sum_{k=1}^{\infty}\frac{\Gamma(k-1/2)}{\Gamma(-k-1/2)}\frac{1}{(4\pi m n\rho)^k}\right] \end{aligned} \tag{3.11}$$

that makes the tree level, one-loop and the non-perturbative instanton contributions manifest.

We use the summation considered in [30, 8] yielding a multi-pole solution of the self-dual Einstein metric by taking $s = -1/2$ yielding

$$F = \sum_{(m,n)=1}\frac{|m\tau+n|}{\sqrt{\mathrm{Im}\tau}} = \sum_{(m,n)=1}\frac{\sqrt{m^2\rho^2+(m\eta+n)^2}}{\sqrt{\rho}} , \tag{3.12}$$

where (m,n) denotes the greatest common divisor. The supersymmetric vacua obtained in gauged supergravity by using the multi-pole solution have been discussed in [8]. The 3-pole solution is particularly interesting, since it will give (after gauging) exactly two supersymmetric vacua and moreover the corresponding family of self-dual Einstein metrics have been discussed in detail already in the literature [30] (and references therein). So, to keep the calculations below simple, we consider this general 3-pole solution expressed in the form,

$$F(\rho,\eta) = \frac{a}{\sqrt{\rho}} + \frac{b+c/q}{2}\frac{\sqrt{\rho^2+(\eta+1)^2}}{\sqrt{\rho}} + \frac{b-c/q}{2}\frac{\sqrt{\rho^2+(\eta-1)^2}}{\sqrt{\rho}} \tag{3.13}$$

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where, a, b, c are some constants and $q^2 = \pm 1$ (in the following we set $q = 1$). As we will see in the next section, already this truncation yields de Sitter vacua in the corresponding gauged supergravity theory. By making this truncation, we seem to have lost the classical limit, i.e. the leading part in the expansion (3.11) (but recall, the series in $s = -1/2$ and $s = 3/2$ are equivalent). This can be justified, because the flow or the critical points are on the line $\rho = 0$, i.e. deep inside the quantum region. But we have to keep in mind that this parameterization also includes the homogeneous quaternionic space as special cases, namely the Bergman metric, which also gives a parameterization of the coset $\mathrm{SU}(2,1)/\mathrm{U}(2)$, is obtained for $a = c = 1$, $b = 0$ and the hyperbolic space $\mathrm{SO}(4,1)/\mathrm{SO}(4)$ corresponds to $b = c < 0$ [30].

4. Gauging the isometries, superpotential and potential

We are primarily interested in de Sitter vacua appearing in gauged supergravity, which may be unstable (saddle points), but the unstable mode should flow into a supersymmetric vacuum. Note, a singular flow is not necessarily related to pole in the potential, also simple run-away potentials yield a singular supergravity solution. Therefore, we are looking for a potential that has two supersymmetric local minima which are connected by a saddle point.

The gauging of the coset space $\mathrm{SU}(2,1)/\mathrm{U}(2)$ has been considered in ref. [7] and it has been shown that as long as no vector multiplets are included the flow will always end in a singularity. In the generic case, one obtains a run-away behaviour, but there are also AdS as well as flat space vacua for this coset.

Let us first see whether the 1-loop correction can give additional supersymmetric vacua, where the function F is given by,

$$F(\rho) = a\rho^{3/2} + \frac{b}{\sqrt{\rho}} \tag{4.1}$$

with a and b as some constants and $b = 0$ corresponds to the classical case. Next, considering a general Killing vector like,

$$k = c_1 \partial_\phi + c_2 \partial_\psi \tag{4.2}$$

one finds that there is again only one supersymmetric fixed point at $\rho^2 = \frac{b}{a}$ and $\eta = -c_2/c_1$. So in this case, any non-supersymmetric de Sitter vacuum is of run-away type or runs into a singularity.

However, considering the instanton corrections as discussed in the last section, one finds that one can have a rich vacuum structure and also good de Sitter vacua which is not the case for the classical and 1-loop corrected UH moduli space. For simplicity, we restrict ourselves to the 3-pole solution involving both ρ and η in a nontrivial way.

Let us now consider gauging a linear combination of the two abelian Killing vectors, namely,

$$k = \beta_1 \partial_\phi + \beta_2 \partial_\psi\ , \tag{4.3}$$

where, β_1 and β_2 are two parameters. The norm of the Killing vector

$$|k|^2 = \frac{1}{F^2 \det Q}(\beta_1\ \beta_2)\, N^t Q^2 N \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} \tag{4.4}$$

has to vanish at fixed points which gives as condition for supersymmetric vacua

$$\det \left| \frac{N^t Q^2 N}{F^2 \det Q} \right| = \frac{1}{F^4} = 0\,. \tag{4.5}$$

For our choice of $F(\rho,\eta)$ in (3.12), this corresponds to $\rho = 0$. But in fact both eigenvalues have to vanish which is the case if: $|m\tau + n| = 0$, i.e. at any given center, see also [30, 8]. For this simple 3-pole solution we have exactly two fixed points at

$$\rho = 0\,, \qquad \eta = \pm 1\,. \tag{4.6}$$

Note, these points are not regular in our coordinate system, but below we will choose a regular coordinate system.

The SU(2) connections for the above CP metric are given by [30]

$$A^1 = -\frac{\partial_\eta F}{F}\, d\rho + \frac{1}{\rho F}\left(\frac{1}{2}F + \rho\partial_\rho F\right) d\eta\,, \tag{4.7}$$

$$A^2 = -\frac{\sqrt{\rho}}{F}\, d\phi\ , \tag{4.8}$$

$$A^3 = \frac{1}{\sqrt{\rho}F}\,(d\psi + \eta d\phi) \tag{4.9}$$

and, with the above Killing vector, the Killing prepotentials are obtained as,

$$P^1 = 0\,, \qquad P^2 = -\beta_1 \frac{\sqrt{\rho}}{F}\,, \qquad P^3 = \frac{1}{\sqrt{\rho}F}\,(\beta_2 + \eta\beta_1)\,. \tag{4.10}$$

The superpotential therefore reads,

$$W \equiv \sqrt{P^i P^i} = \frac{\sqrt{\beta_1^2 \rho^2 + (\beta_2 + \eta\beta_1)^2}}{\sqrt{\rho}F}\,. \tag{4.11}$$

The different supersymmetric vacua have been discussed already in [8], see also [34] for a different parameterization of the 3-pole solution. In this paper we are interested in de Sitter vacua emerging between two supersymmetric vacua, which is possible if both supersymmetric vacua are local minima of the supergravity potential V. By definition, a deSitter vacuum corresponds to a *positive* extremum of the potential (see (4.18)), i.e.

$$dV = 0\,, \qquad V_0 > 0\,. \tag{4.12}$$

In the generic situation in gauged supergravity, deSitter vacua represent saddle points, i.e. are unstable at least in one direction. In our case, the potential depends only on two scalar fields and to make sure that the value of the potential at the saddle is positive, we choose one of the supersymmetric vacua as flat space. Note, a supersymmetric flat space vacuum has a positive definite mass matrix (at least as long the moduli metric does not become degenerate at this point) and hence, choosing one supersymmetric vacuum as flat space, there has to be a deSitter saddle point in-between. This requires of course, that the potential is also regular between the supersymmetric vacua, which we will show below.

If we take $\beta_1 = 1$ and $\beta_2 = -1$ for the Killing vector, one obtains the superpotential as

$$W = \frac{\sqrt{\rho^2 + (\eta - 1)^2}}{\sqrt{\rho} F} . \tag{4.13}$$

which then vanishes at the critical point $\rho = 0$, $\eta = 1$.

Let us now introduce new coordinates

$$\rho = \sinh r \, \cos\theta \, , \qquad \eta = \cosh r \, \sin\theta \, . \tag{4.14}$$

giving

$$\begin{aligned} \frac{1}{4\rho}\left[F - 4\rho^2(F_\rho^2 + F_\eta^2)\right] &= \frac{b^2 - c^2 + a(b\cosh r - c\sin\theta)}{\cosh^2 r - \sin^2\theta} \\ \sqrt{\rho}F &= a + b\cosh r + c\sin\theta \, . \end{aligned}$$

For the flow equations, only the (ρ, η) part of the metric matters and in the new coordinates, this part becomes

$$\frac{c^2 - b^2 - a(b\cosh r - c\sin\theta)}{(a + b\cosh r + c\sin\theta)^2}\left(dr^2 + d\theta^2\right) . \tag{4.15}$$

We have changed here the sign of the metric as we are interested in negatively curved anti-selfdual Einstein space, see discussion after eq. (3.4). In these coordinates, the superpotential becomes

$$W = \frac{\cosh r - \sin\theta}{a + b\cosh r + c\sin\theta} \tag{4.16}$$

yielding fixed points at

$$r = 0 \, , \qquad \sin\theta = \pm 1 \, . \tag{4.17}$$

So, we see that θ can run between a flat space vacuum $(r = 0, \theta = \pi/2)$ where $W = dW = 0$ and an AdS vacuum $(r = 0, \theta = -\pi/2)$ where $W \neq 0$, $dW = 0$. Note, with this change of coordinates the metric at the fixed points is well defined, but there is still a coordinate singularity at $\sqrt{\rho}F = a + b\cosh r + c\sin\theta = 0$, which represents a pole in the superpotential and hence the supergravity potential V is not bounded from below. But by fixing the parameter appropriately this pole will be separated by a potential barrier. On the other hand, the moduli space has a singularity at $\det Q = 0$, which in the new coordinates corresponds to the point where $c^2 - b^2 - a(b\cosh r - c\sin\theta) = 0$. Recall that the sign of $(\det Q)$ fixes the sign of the curvature of the 4-dimensional space and hence we have to restrict ourselves to regions where $\det Q < 0$ and drop the overall sign in the metric (this is what we have done already in (4.15)).

In order to investigate the possibility of de Sitter vacua, we have to discuss the supergravity potential, which is given by

$$V = 2\left[g^{rr}(\partial_r W)^2 + g^{\theta\theta}(\partial_\theta W)^2 - \frac{3}{2}W^2\right] . \tag{4.18}$$

At the two BPS extrema ($r = 0$, $\theta = \pm\pi/2$), the eigenvalues of the hessians of W and V are given by,

$$
\begin{aligned}
ddW\Big|_{\theta=\frac{\pi}{2}} &= \left[\frac{1}{a+b+c}, \frac{1}{a+b+c}\right] \\
ddV\Big|_{\theta=\frac{\pi}{2}} &= 4\left[\frac{1}{(c-b)(a+b+c)}, \frac{1}{(c-b)(a+b+c)}\right] \\
ddW\Big|_{\theta=-\frac{\pi}{2}} &= \left[\frac{a-b-c}{(a+b-c)^2}\ ,\ -\frac{a+b+c}{(a+b-c)^2}\right] \\
ddV\Big|_{\theta=-\frac{\pi}{2}} &= 4\left[\frac{(2[b+c]+a)(b+c-a)}{(b+c)(a+b-c)^3}, \frac{(2[b+c]-a)(a+b+c)}{(b+c)(a+b-c)^3}\right]
\end{aligned}
\tag{4.19}
$$

with $W|_{\theta=-\frac{\pi}{2}} = \frac{2}{a+b-c}$. As imposed by supersymmetry, the masses of the scalars are the same in the flat space vacuum at ($r = 0$, $\theta = \pi/2$).

As one might have expected for this inhomogeneous space, the metric by itself is not positive definite, there are different regions in parameter space where the metric has the correct signature — one of them will contain the singularity. We now want to choose the parameter in a way, that the region containing both supersymmetric fixed points, is regular and has the correct signature. Therefore, we have to impose the following relations

$$
c^2 - b^2 - a(b \pm c) = -(b \pm c)(a + b \mp c) > 0\,, \qquad a + b \mp c < 0 \tag{4.20}
$$

yielding $b \pm c > 0$ or

$$
c^2 - b^2 - a(b \pm c) = -(b \pm c)(a + b \mp c) > 0\,, \qquad a + b \mp c > 0\,, \tag{4.21}
$$

giving $b \pm c < 0$. The first condition ensures that the metric (4.15) at both BPS fixed points is positive definite and the second condition implies that there is no coordinate singularity in-between, i.e. $\sqrt{\rho}F$ does not change its sign. It is straightforward to verify that these conditions also ensure that the flat space is a local minimum. To simplify the notation further let us set $b = \pm 1$ which gives the new relations

$$
\begin{aligned}
b &= 1\,, \qquad & a &< -2\,, \qquad & |c| &> 1\,, \\
b &= -1\,, \qquad & a &> 2\,, \qquad & |c| &< 1\,.
\end{aligned}
\tag{4.22}
$$

In the figure 1 we have plotted an example for the potential, where the BPS vacua are local minima and in-between is a deSitter saddle point (it is of course 2π-periodic in θ). In the flat space vacuum the mass matrix is always positive definite, but in the anti deSitter vacuum $\partial_r\partial_r V$ is positive only if $a > 2|b+c|$ (note $b+c < 0$ in this example), but even if this does not hold, this vacuum is stable due to the Breitenlohner-Freedman bound saturated by any BPS fixed point. Moreover, we see that the critical line $r = 0$ is a local minimum, but for large r we always run into a pole of the superpotential, where

$$
a + b\cosh r + c\sin\theta = 0
$$

(this point in moduli space is smooth, but it represents the endpoint of a given coordinate region). In the supergravity potential V, a pole in W can result in a positive or negative pole depending on whether the first term grows faster than the second. Unfortunately, in

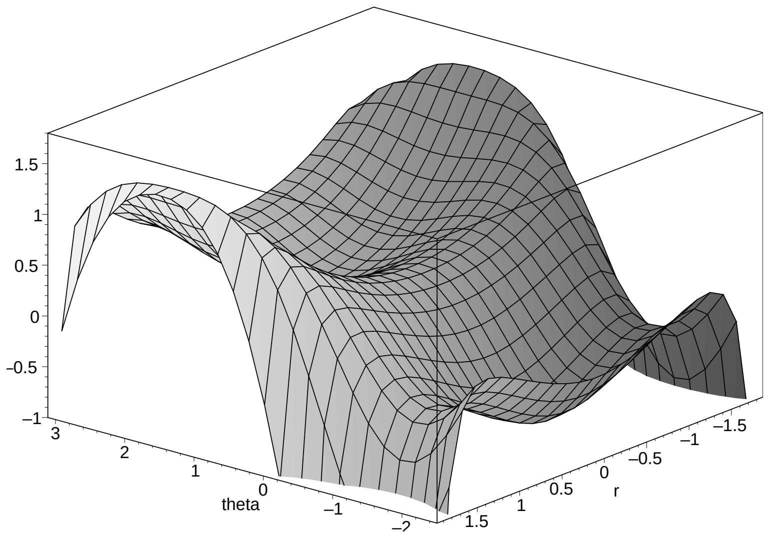

Figure 1: For this plot we have chosen $a = 5$, $b = -1$, $c = 0.1$. It shows one AdS minimum at $(r, \theta) = (0, -\pi/2)$, a flat space vacuum at $(r, \theta) = (0, \pi/2)$ and in-between there is a deSitter saddle point.

our case it is negative pole, which can be seen as follows. In addition to the critical line: $r = 0$, also the lines: $\theta = \pm\pi/2$ are critical (i.e. $\partial_\theta V|_{\theta=\pm\pi/2} = 0$ for all values of r). Let us now investigate the line $\theta = \pi/2$, where the potential around the flat space vacuum is positive definite and vanishes at $r = 0$. It is a straightforward exercise to see that the potential along this line as function of $\cosh r$ has two further zeros at

$$\cosh r_\pm = -\frac{1}{6ab}\Big[5b^2 - c^2 + 2a^2 + a(c+b) + 4cb \pm \\ \pm \sqrt{(25b^2 + 4a^2 + c^2 - 28ab - 10cb - 4ac)(c+a+b)^2)}\Big] . \quad (4.23)$$

The r.h.s. is exactly equal to one at: $a + b + c = 0$, but for all other values discussed in eqs. (4.20) or (4.21), only one of the solutions obeys: $\cosh r_\pm > 1$. This implies, that the potential $V|_{\theta=\pi/2}$ has, apart from the point at $r = 0$, always one additional zero. Hence, there is a *negative* pole in V and there is another de Sitter vacuum, where V reaches its maximum. The same analysis can be repeated for the critical line at $\theta = -\pi/2$. If the BPS vacuum at $r = 0$ is a local minimum we get another de Sitter saddle point at the edge of the negative pole. Note, the existence of these two additional de Sitter extrema is a simple consequence of the positivity of the hessian of V at each BPS extremum and the appearance of the negative pole in the potential for large values of r.

Recall that the appearance of *two* BPS extrema was a consequence of the 3-pole ansatz in (3.13) and each additional pole results into another supersymmetric extremum, but of course not all of them are connected, i.e. they should be separated by poles of the superpotential. In fact as we know from the RG-flow discussion, smoothly connected BPS flows are only possible between the so-called UV (ultraviolet) and IR (infrared) or IR and

IR fixed points, see e.g. [7]. All BPS fixed points are on the line $r = 0$ and are separated in the η-direction and therefore the η-dependence was crucial in our setup — the 1-loop correction to the classical result is not sufficient.

Let us end this section, with a discussion for the case when both BPS vacua are AdS, which is the case if we choose for the Killing vector: $\beta_1 = 1$ and $\beta_2 = \lambda \neq \pm 1$. The superpotential for such case in r and θ coordinates is given by,

$$W = \frac{\sqrt{-\cos^2\theta + \cosh^2 r + 2\cosh r \sin\theta\lambda + \lambda^2}}{a + b\cosh r + c\sin\theta} \,. \tag{4.24}$$

The BPS critical points are again at $r = 0$ and $\cos\theta = 0$ which gives

$$\begin{aligned} W\Big|_{r=0,\theta=\frac{\pi}{2}} &= \frac{\lambda+1}{a+b+c} \\ W\Big|_{r=0,\theta=-\frac{\pi}{2}} &= \frac{\lambda-1}{a+b-c} \end{aligned} \tag{4.25}$$

and for $\lambda = \pm 1$ we get back to our previous case of flat space and AdS vacuum. Choosing $|\lambda| > 1$, one fixed point is of UV and the other is of IR type and for $|\lambda| < 0$ both are of IR type, which is the relevant setup for the Randall-Sundrum scenario [4]. Since the r direction is stable ($\partial_r\partial_r V > 0$), one can consider the case where the scalar r is frozen to its fixed point value. One then gets,

$$W = \frac{\lambda + \sin\theta}{a + b + c\sin\theta} \,. \tag{4.26}$$

The potential V is then given by,

$$V = 2\left[\frac{\cos^2\theta\,(a+b-c\lambda)^2}{[c^2-b^2-a(b-c\sin\theta)](a+b+c\sin\theta)^2} - \frac{3}{2}\frac{(\lambda+\sin\theta)^2}{(a+b+c\sin\theta)^2}\right] . \tag{4.27}$$

Now, with the choice of parameter as in (4.22), the potential V as well as the superpotential W are regular for all values of θ and V has to have another extremum between both supersymmetric vacua at: $\sin\theta = \pm 1$. Depending on λ the potential at this extremum can be positive (deSitter) or negative (anti-deSitter). For $\lambda = 0$, the superpotential W has a zero at $\theta = 0$ resulting in a positive potential V (deSitter maximum) and for the case discussed before [with $\lambda = -1$], the maximum is also a deSitter vacuum. E.g. for the parameter choosen in figure 1 one finds $\partial_\theta V = 0$ at $\sin\theta \simeq 0.213$, where $V_0 \simeq 0.367$.

5. Cosmological flow towards the flat space minimum

Let us go back to the case where we have the flat space and AdS vacua as the two supersymmetric extrema (for the Killing vector with the choice $\beta_1 = 1$ and $\beta_2 = -1$). As we saw before, the non supersymmetric vacuum corresponding to the de Sitter saddle point had an unstable direction along θ. Since the solution is stable along the r direction, we can freeze the r coordinate to its fixed point value so that there is only one scalar field θ. One can now look for a time dependent solution departing from the de Sitter extremum along the θ direction to reach the supersymmetric flat space minimum. For this, we have to consider the scalar field θ having a time dependence and we make an ansatz for a time dependent flat Robertson-Walker metric in four dimensions, which is given by,

$$ds^2 = -dt^2 + e^{2a(t)}\left(dx^2 + dy^2 + dz^2\right), \tag{5.1}$$

where $a(t)$ is the scale factor.

If one now starts at the de Sitter saddle point and runs towards the AdS minimum, then with the above metric, the system can never settle down in this BPS vacuum, which is a consequence of the Einstein equations and has been used in the fast roll inflation proposal [22], see also [35]. However, one has to keep in mind that this conclusion is a consequence of the metric ansatz and allowing for a non-flat spatial section and/or allowing for a dependence on the spatial coordinates, the system may settle down in the AdS minimum. But the above metric ansatz is phenomenologically most interesting and hence let us stick to this ansatz.

The action involving the scalar field θ and the potential $V(\theta)$ is given by,

$$S \sim \int dt d^3x\sqrt{-g}\left(R - \frac{1}{2}g_{\theta\theta}g^{\mu\nu}\partial_\mu\theta\partial_\nu\theta - V(\theta)\right), \tag{5.2}$$

where, μ, ν are 4-dimensional space-time indices. The equations of motion become

$$\begin{aligned}
3\dot{a}^2 - \frac{1}{4}g_{\theta\theta}\dot{\theta}^2 - \frac{1}{2}V(\theta) &= 0\\
-2\ddot{a} - 3\dot{a}^2 - \frac{1}{4}g_{\theta\theta}\dot{\theta}^2 + \frac{1}{2}V(\theta) &= 0\\
g_{\theta\theta}(\ddot{\theta} + 3\dot{a}\dot{\theta}) + \partial_\theta V(\theta) + \frac{1}{2}(\partial_\theta g_{\theta\theta})\dot{\theta}^2 &= 0,
\end{aligned} \tag{5.3}$$

where the last equation is the scalar equation of motion. Adding the first two equations, one gets,

$$\ddot{a} = -\frac{1}{4}g_{\theta\theta}(\dot{\theta})^2$$

which is also known as the cosmological c-theorem (i.e. $\dot{a}$ is a monotonic function). Then differentiating the first equation with respect to time, one gets,

$$6\dot{a}\ddot{a} - \frac{1}{2}g_{\theta\theta}\dot{\theta}\ddot{\theta} - \frac{1}{4}\partial_\theta g_{\theta\theta}(\dot{\theta})^3 - \frac{1}{2}\dot{\theta}\partial_\theta V = 0. \tag{5.4}$$

Inserting the value of $\partial_\theta V$ from the scalar field equation, and using $\ddot{a} = -\frac{1}{4}g_{\theta\theta}(\dot{\theta})^2$ in the above equation, one finds that one of the metric equation is redundant. Hence one has to solve only the scalar equation and the g_{00} equation to obtain time dependent solutions for θ and the scale factor. Note that, these two equations do not reduce to first order equations as has been the case for the BPS domain wall solutions. Though we have not solved these equations explicitly, qualitatively one finds that the solution asymptotes to the flat space where $W = dW = 0$ which means that the scalar field rolls down from the de Sitter saddle point and stabilizes at the minimum.

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Of course the appearance of the negative pole in the potential questions the model from the phenomenological perspective. It would have been much better, if one had found a parameterization so that V was bounded from below. Recall, poles in the superpotential can also yield positive poles in V for the case that first term grows faster than the second. Unfortunately, we were not able to find such a setup. On the other hand, one can of course identify the Big Bang with this pole and with the correctly tuned initial velocity, the scalars can reach the de Sitter saddle point and finally settle down in the flat space vacuum.

6. Discussion

In this paper, we have studied the gauging of the isometries of the quantum corrected universal hypermultiplet moduli space, where the corresponding quaternionic geometry has been given by the Calderbank - Pedersen metric. We have analyzed the superpotential and the potential to obtain the fixed points relevant in the context of holographic RG flow. We have considered the simplest 3-pole solution which essentially includes the non-perturbative instanton corrections and we have explored the possibility of obtaining de Sitter vacua with such a solution. We have found a de Sitter saddle point that connects two supersymmetric minima of the potential. Moreover, the potential has one de Sitter maximum as well as another de Sitter saddle point, which are however at the edge of a negative pole in the potential.

The dS/CFT correspondence [36] has led to the conjecture that the Universe is an RG flow between two conformal fixed points of a three dimensional Euclidean field theory where the time evolution in the bulk has been interpreted as inverse RG flow in the dual CFT [37]. In ref.[35], the cosmological evolution of a scalar field coupled to gravity has also been studied and a detailed analysis of the extrema of the potential and the flow equations have been done. Our case is different from these considerations as the flow considered here does not really have a field theory interpretation. Note that the flow from the de Sitter saddle point to the AdS vacua will be ruled out as the (time-dependent) scalars can not settle in the AdS minimum and only a flow towards flat space minimum is allowed (at least as long as one sticks to the spatially flat Robertson-Walker metric). We also should mention here that if we consider the case where we have two AdS supersymmetric vacua, then with the above metric ansatz, there should be only bubble solution where starting from the dS vacuum, one runs to the another dS vacuum which will be more in line with the dS/CFT correspondence.

Finally we would like to comment on the relevance of our superpotential to the recently discussed new-old inflationary models [23] where the potential does not have to satisfy the slow roll condition. There, it has been possible to get inflation when the system is locked at a saddle point by sufficient oscillation along the stable direction and eventually the scalar rolls down to the true minimum of the potential. It might be interesting to see whether the de Sitter saddle point can be used for a locked inflation before the system settles down in the flat space minimum. The scalar potential depending on two scalar fields r and θ, when expanded up to second order and choosing for example, the constant parameters a, b, c satisfying the condition to avoid the curvature singularity, one can put it into a form

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as in the new old inflationary scenario. Our potential has however no false vacuum and hence, from the cosmological point of view, the evolution can start with the singularity corresponding to the negative pole in the potential. But then in order to reach the saddle point and initiate a locked inflation, scalar fields have to have fine-tuned initial velocities. A more detailed investigation along these lines will be very interesting.

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